Fast and Accurate Measurement of Linear Transducer Parameters

W. Klippel, U. Seidel
KLIPPEL GmbH
Germany
www.klippel.de

ABSTRACT

A new measurement technique is presented for the estimation of the linear parameters of the lumped transducer model. It is based on the measurement of the electrical impedance and the voice coil displacement using a laser sensor. This technique identifies the electrical and mechanical parameters directly and dispenses with a second measurement of the driver using a test enclosure or an additional mass. Problems due to leakage of the enclosure or the attachment of the mass are avoided giving accurate and reliable results. The measurement of the displacement also allows identification of the mechanical compliance versus frequency (explaining suspension creep) which is the basis for predicting the radiated sound pressure response at low frequencies precisely. The linear parameters measured at various amplitudes are compared with the results of large signal parameter identification and the need for nonlinear transducer modelling is discussed.

INTRODUCTION

The determination of linear loudspeaker parameters belongs to the classical problems in driver design. It can be solved straight forward and computer programs that calculate the linear parameters are available for years. Are there any news?

Traditionally the impedance function is measured and analyzed. The impedance is in fact a good basis for linear parameter identification. It is easy to measure, it is not affected by the acoustic system and does not contain any time delay. Unfortunately it describes only the electrical part of the model and additional information from the mechanical domain is required to determine the mechanical parameters. Therefore, a second measurement is performed where the transducer is either mounted in a test enclosure or an additional mass is attached to it (perturbation method). Apart from being time consuming the accuracy of the results may be deteriorated by leakage of the enclosure and problems due to the attachment of the mass. There are even transducers for which neither of the techniques can be applied.

In the paper an identification technique is proposed that dispenses with a second measurement. The problems mentioned above are avoided giving accurate and reproducible results. Furthermore a procedure is described to validate the identification results. The linear parameters describe the loudspeaker adequately only if the excitation is sufficiently small. They fail to describe the large signal behavior of the speaker. The behavior at high amplitudes and the relationship between linear and nonlinear parameters are discussed in the second part of the paper.

TRANSDUCER MODEL

The paper presents a novel technique to identify the components (Thiele-Small Parameters) of the linear loudspeaker model below valid in the small signal domain.

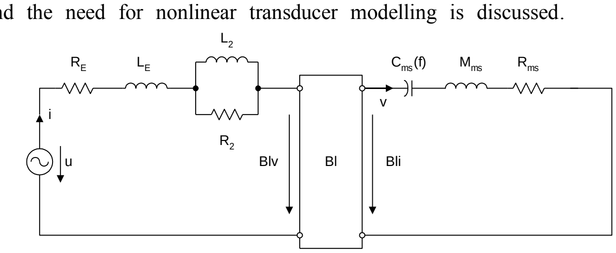

Fig.1: Linear loudspeaker model

Electrical Parameters	
R_E	electrical voice coil resistance at DC
L_E	voice coil inductance at low frequencies
L_2	para-inductance at high frequencies
R_2	resistance due to eddy currents
Derived Parameters	
$C_{MES} = M_{MS} / B^2l^2$	electrical capacitance representing mechanical mass
$L_{CES} = C_{MS}\ B^2l^2$	electrical inductance representing mechanical compliance
$R_{ES} = B^2l^2 / R_{MS}$	resistance due to mechanical losses
f_s	driver resonance frequency
Mechanical Parameters	
M_{MS}	mechanical mass of driver diaphragm assembly including air load and voice coil
R_{MS}	mechanical resistance of total-driver losses
K_{MS}	mechanical stiffness of driver suspension
$C_{MS} = 1 / K_{MS}$	mechanical compliance of driver suspension
Bl	force factor (*Bl* product)

In contrast to the large signal model we assume that all parameters of the lumped elements are independent of the state variables.

IDENTIFICATION ALGORITHM

In order to identify the electrical and mechanical parameters of the linear loudspeaker model (figure 1) a multi tone test signal is applied to the loudspeaker. A voltage and a current sensor are required to measure voltage *u(t)* and current *i(t)* at speaker terminals. Furthermore the diaphragm displacement *x(t)* was measured using a laser displacement sensor based on geometrical triangulation. The whole measurement setup can be seen in figure 2.

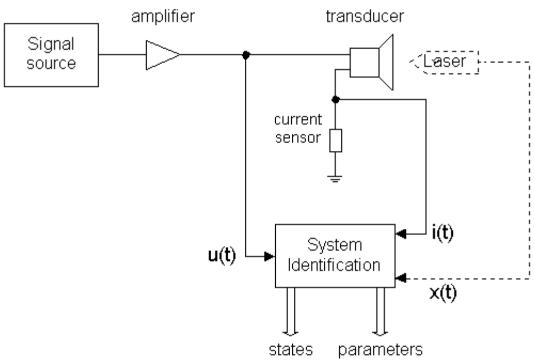

Fig. 2: Measurement setup

Electrical Parameters

The electrical parameters are determined by calculating the spectra *U(s)*, *I(s)* of voltage *u(t)* and current *i(t)*, respectively, and exploiting the electrical impedance $Z(s)=U(s)/I(s)$. According to the linear loudspeaker model the electrical parameters and the impedance are related by

$$Z(s)=\frac{U(s)}{I(s)}=\frac{sL_{CES}}{s^2L_{CES}C_{MES}+s\dfrac{L_{CES}}{R_{ES}}+1}+\frac{sL_2R_2}{sL_2+R_2}+sL_E+R_E$$

Thus the least squares algorithms can be applied to determine the electrical parameters by fitting the right hand side of the above equation to the measured impedance function. Figure 3 shows the measured and the fitted impedance curve for a real speaker. Note that no laser is required to determine the electrical parameters.

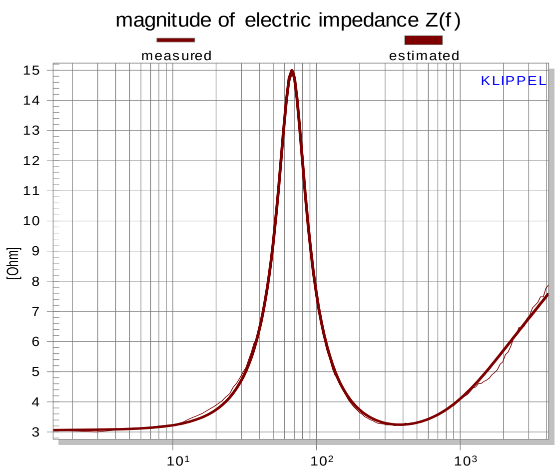

Fig. 3: Measured and estimated impedance

Force Factor and Mechanical Parameters

Traditional techniques for the estimation of the mechanical parameters require a second (perturbed) measurement where the transducer is either mounted in a test enclosure or an additional mass is attached to it. Both techniques are time consuming and the accuracy of the results may be deteriorated by leakage of the enclosure and problems due to the attachment of the mass. There are also transducers neither of the techniques can be applied.

Using the displacement signal *x(t)* the force factor *Bl* can be calculated at one swoop. Consider the transfer function terminal voltage to displacement

$$H_x(s)=\frac{X(s)}{U(s)}=\frac{X(s)Bls}{U(s)}\cdot\frac{1}{Bls}$$
$$=\frac{Z_R}{Z_R+Z_P+sL_E+R_E}\cdot\frac{1}{Bls}$$

where *U(s)*, *X(s)* denote the spectra of terminal voltage and displacement respectively and

$$Z_R(s)=\frac{sL_{CES}}{s^2L_{CES}C_{MES}+s\dfrac{L_{CES}}{R_{ES}}+1}$$

$$Z_P(s)=\frac{sL_2R_2}{sL_2+R_2}\,.$$

Note that after identifying the electrical parameters the transfer function $H_x(s)$ is completely except for the linear factor (*Bl*-product) in the equation above. It is thus straightforward to apply the least squares method to determine *Bl*. However fitting $H_x(s)$ to the measured transfer function *X(s)/U(s)* will normally produce insufficient results as can be seen in figure 4. The estimated and measured transfer function $H_x(s)$ differ considerably at low frequencies. This discrepancy can not be eliminated by adjusting *Bl*. *Bl* determines the vertical shift but does not affect the shape of the estimated curve. In order to overcome the problem the model has to be refined.

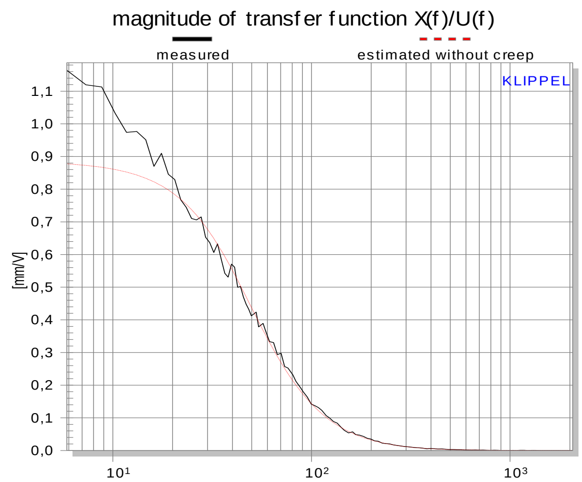

Fig. 4: Transfer function H_x (measured/estimated without creep model)

Creep Model

The mechanical suspension exposed to a sustained force will show varying displacement versus time. This is commonly referred to as creep effect. The time dependence of the stiffness can not be described by a static model. Instead a dynamic model is required leading to a frequency dependence of the stiffness. Usually the stiffness becomes smaller at lower frequencies.

In order to model the compliance $C_{MS}(j\omega)$ as a frequency varying parameter we followed a proposal of Kundsen and Jensen [1]. We replaced the constant compliance by the dynamic transfer function

$$C_{MS}(j\omega) = C_{MS}\left[1 - \lambda \cdot \log_{10}\left(\frac{j\omega}{j\omega_s}\right)\right], \omega_s = 2\pi f_s$$

where C_{MS} denotes the linear compliance and f_s is the driver resonance frequency. There is a straightforward interpretation of the suspension creep factor λ. The quantity $\lambda \cdot 100\%$ indicates the increase of the linear compliance C_{MS} in percentages at low frequencies. For a frequency one decade below the resonance frequency f_s the linear compliance C_{MS} is increased by $\lambda \cdot 100\%$ percent.

As $C_{MS}(j\omega)$ depends linearly on λ the least squares method can be applied to determine the creep factor λ. Using the extended model we were able to get a good agreement between measured and estimated transfer function H_x (figure 5, λ=0.376) for the driver of figure 4.

Note that the impedance function is not capable for giving sufficient information about the suspension at low frequencies.

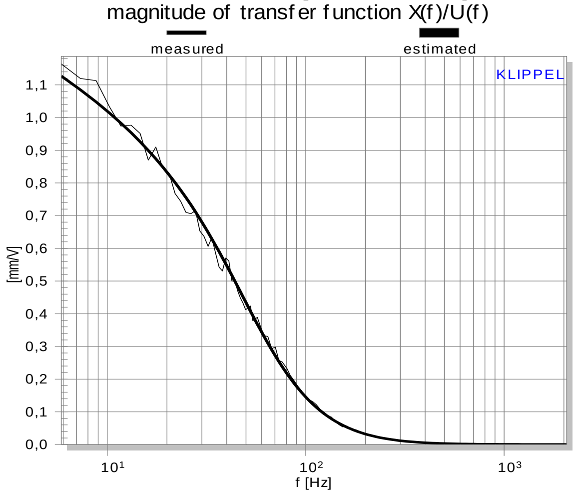

Fig. 5: Transfer function H_x (measured / estimated with creep model)

Once the force factor *Bl* is identified the other mechanical parameters can be calculated using *Bl* and the electrical parameters.

Sound Pressure Prediction

Using the estimated transfer function $H_x(s)$ the sound pressure in the far field can be predicted easily. Assuming a radiation in a half space (2π-sr free field) the following relation

$$P(t) = \frac{d^2X(t)}{dt^2} \cdot \frac{S_D \rho}{2\pi r}$$

holds where P, S_D, r and ρ denote the sound pressure, diaphragm area, distance and density of air. With $X(s)=H_x(s)U(s)$ this corresponds to

$$P(s) = s^2 H_x(s) U(s) \cdot \frac{S_D \rho}{2\pi r}$$

in the frequency domain. Figure 6 shows the estimated sound pressure spectrum for a real driver.

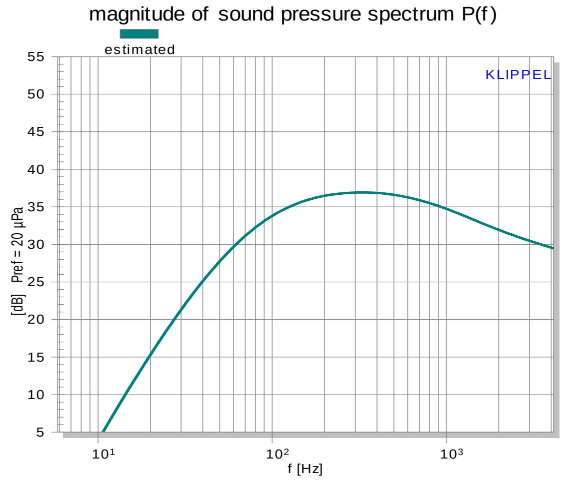

Fig. 6: Estimated sound pressure spectrum

ENSURING VALIDITY OF THE IDENTIFICATION RESULTS

Any algorithm that identifies loudspeaker parameters will produce valid results only under certain measurement conditions. It is therefore crucial to check whether the required conditions could be kept during the measurement. Identified loudspeaker parameters are worthless without proper validity check. The signals measured at the driver contain information which can be exploited to detect automatically

- disconnected sensors,
- amplifier limiting,
- insufficient signal to noise ratio and
- driver working beyond linear range.

In the following the validation procedure is discussed in detail.

Current Signal

The validity of the current sensor signal can be checked with the plot that can be seen in figure 7. It shows the spectral lines of the current signal, the noise floor and lines that correspond to the noise and distortions generated by the speaker. Note the notch of the spectra at the resonance frequency of the loudspeaker. The current signal is invalid if:

1. The signal lines are not well above the noise floor (signal to noise ratio is to low). The current signal is noise corrupted. Remedy: Increase of excitation signal amplitude and/or increase of number of averaging.

2. The difference noise+distortions and noise floor is not negligible (like in figure 7). The current signal is

distorted by the speaker's nonlinearities. A high amount of distortion in the current indicates that the linear model is not adequate anymore. Remedy: Reduction of excitation signal amplitude.

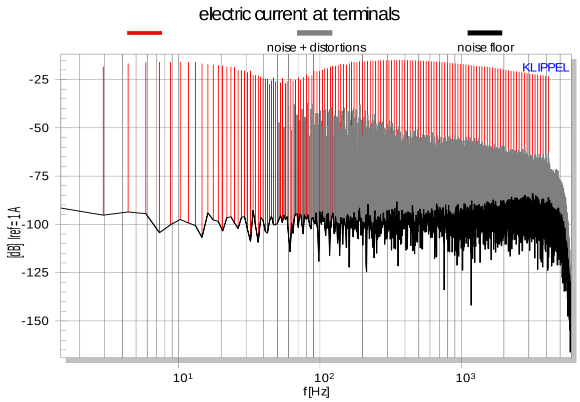

Fig. 7: Spectrum of the current sensor signal

Voltage Signal

Figure 8 shows the plot that is used to check the validity of the voltage signal. The voltage signal is invalid if:

1. The signal lines are not well above the noise floor (signal to noise ratio is to low). Remedy: Increase of excitation signal amplitude and/or increase of number of averaging.

2. The difference noise+distortions and noise floor is not negligible (like in figure 8). The voltage signal is distorted by the amplifiers nonlinearities such as amplifier limiting. Remedy: Reduction of excitation signal amplitude.

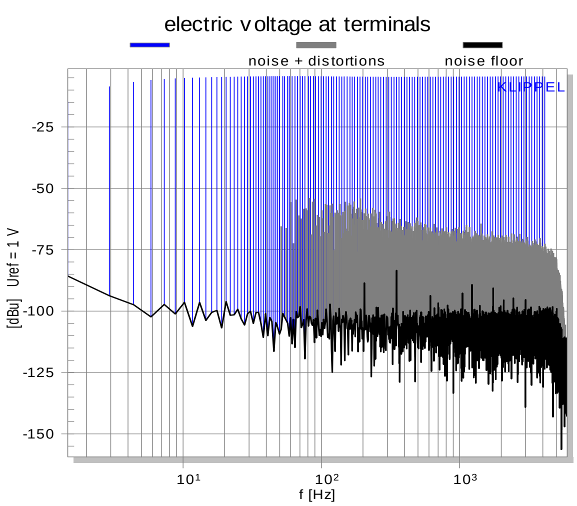

Fig. 8: Spectrum of the voltage sensor signal

Displacement Signal

Figure 9 shows the plot that is used for validating the laser displacement sensor signal. The displacement spectra will decay with 12 dB above the resonance frequency of the laser. The displacement signal is invalid if:

1. The frequency where the displacement spectra disappears in the noise is lower than 300 Hz. Remedy: Increase of excitation signal amplitude and/or increase of number of averaging.

2. There are only a few signal lines well above the noise floor. That might be caused by a high resonance frequency of the loudspeaker that is too close to the cut-off frequency of the laser head. Remedy: Increase of excitation signal amplitude and/or increase of number of averaging.

3. The difference noise+distortions and noise floor is not negligible (like in figure 9). The voice coil displacement is distorted by the speaker's nonlinearities. A high amount of distortion in the displacement indicates that the linear model is not adequate anymore. Remedy: Reduction of excitation signal amplitude.

4. The difference noise+distortions and noise floor is not negligible. The output of the sensor is distorted due to nonlinearities inherent in the triangulation principle (used to measure the distance between laser head and diaphragm). Remedy: Increase of excitation signal amplitude.

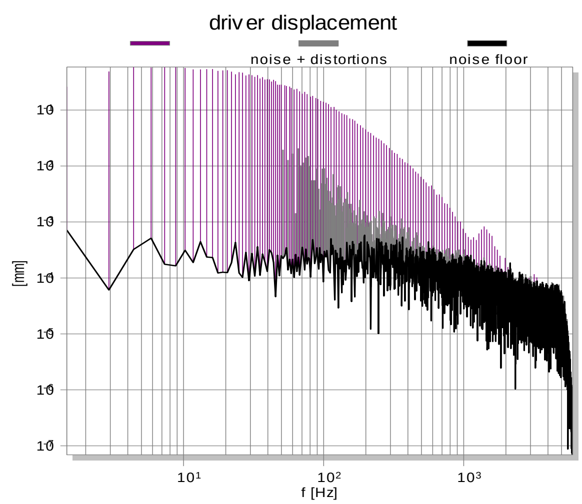

Fig. 9: Spectrum of the laser sensor signal

Accuracy of the measurement

To check the reliability of the measured linear parameters the measurement has been repeated ten times under identical conditions. After subjecting the data to a statistical analysis the results are presented in Table I.

Parameter	**Mean Value**	**Standard Deviation σ**	**unit**	**Deviation in Percent**
R_E	3.02	0.0092	Ω	0.3003
L_E	0.132	0.0007	mH	0.4219
L_2	0.264	0.0016	mH	0.7077
R_2	3.52	0.0183	Ω	0.5723
C_{MES}	433	1.5635	µF	0.3377
L_{CES}	11.75	0.0467	mH	0.3177
R_{ES}	12.08	0.0259	Ω	0.2352

f_s	70.5	0.0568	Hz	0.0931
M_{MS}	11.87	0.0750	g	0.7041
R_{MS}	2.27	0.0132	kg/s	0.6305
C_{MS}	0.43	0.0057	mm/N	0.8883
K_{MS}	2.33	0.0108	N/mm	0.6902
Bl	5.23	0.0178	N/A	0.3704
λ	0.376	0.0078		5.0760

Table I: Reproducibility of the Linear parameters

Almost all of the parameters vary less than 1 % from the mean. Only the creep factor λ shows a significant higher deviation about 5 % which may be caused by the time-varying properties of the suspension during test. Thus, using an inexpensive laser head does not only expedite the measurement but gives much more reproducibility than measurements based on perturbation methods.

ABUSING THE LINEAR MODEL

The linear loudspeaker model (figure 1) is adequate only if the amplitude of the excitation is sufficiently small. What happens if this condition is violated ? In the table below the identification results for different amplitudes of excitation are listed. Note that only the bold parameters are valid. The table shows that the parameters vary considerable as the amplitude is increased.

Displacement (peak value)	**Small (0.17 mm)**	**Medium (1.08 mm)**	**Large (8.57 mm)**	
R_E	**3.02**	3.07	3.11	Ω
L_E	**0.132**	0.136	0.157	mH
L_2	**0.264**	0.262	0.274	mH
R_2	**3.52**	3.66	3.95	Ω
C_{MES}	**433**	445	496	µF
L_{CES}	**11.75**	14.57	15.67	mH
R_{ES}	**12.08**	10.76	9.68	Ω
f_s	**70.5**	62.5	57.1	Hz
M_{MS}	**11.87**	12.35	12.21	g
R_{MS}	**2.27**	2.58	2.54	kg/s
C_{MS}	**0.43**	0.52	0.64	mm/N
K_{MS}	**2.33**	1.91	1.57	N/mm
Bl	**5.23**	5.27	4.96	N/A

Table II: Linear Parameters measured at different amplitudes

The compliance C_{MS} increases by 50 % reducing the resonance frequency by 13 Hz. Surprisingly the results indicate that the suspension becomes softer while exposed to higher amplitudes. However, pushing the suspension by hand up to mechanical limits we feel that the suspension gets very stiff beyond a certain displacement. This contradiction, which can be observed on many drivers, shows that some extension of the linear model is required to explain the driver's behaviour in the large signal domain.

Relationship To Large Signal Parameters

Whereas the linear model assumes that all parameters are constant, large signal modeling considers the dependence of the parameters on the driver's instantaneous state variables such as voice coil temperature and displacement. Above all the inductance parameter $L_E(x)$, the force factor $Bl(x)$ and the stiffness $K_{MS}(x)$ vary substantially with the instantaneous displacement x causing audible distortion. The nonlinear curves can be measured dynamically by monitoring the electrical terminal signals and applying system identification techniques (Distortion Analyzer).

Figure 10 shows the *Bl*-product as a function of displacement *x*. At maximal displacement x_{peak} = ± 9.5 mm (coil maximal in and out) the force factor is only 25 % of the value at the rest position *Bl(0)*=5.2. Since the *Bl*-product determines the driving force and the electrical damping of the mechanical system, high *Bl*(x)-variations produce not only significant harmonic distortion at low frequencies but may also produce high magnitude broad-band intermodulations between a low frequency component (bass) and a high-frequency component (voice).

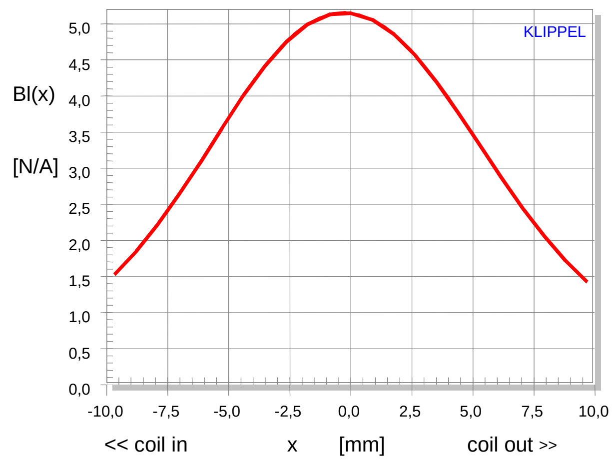

Fig. 10: *Bl*-product versus voice coil displacement *x*

The *Bl*(x)-characteristic affects the electrical damping of the speaker dramatically. Figure 11 shows the loss factor $Q_{ES}(x)$ versus displacement x considering the electrical losses only. If the Bl(x) reduces to 25 percent the electrical loss factor $Q_{ES}(x)$ will increase by factor 16 because $Q_{ES}(x)$ is a function of $Bl^2(x)$.

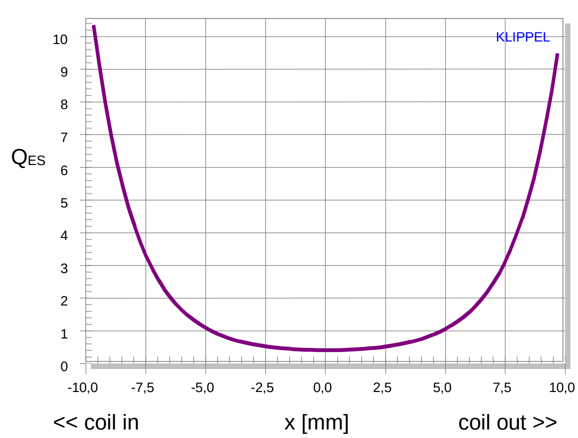

Fig. 11: Electrical loss factor $Q_{ES}(x)$ versus displacement

If the electrical damping vanishes the remaining mechanical damping represented by Q_{MS} will determine the total $Q_{TS}(x)$ as shown in Fig 12.

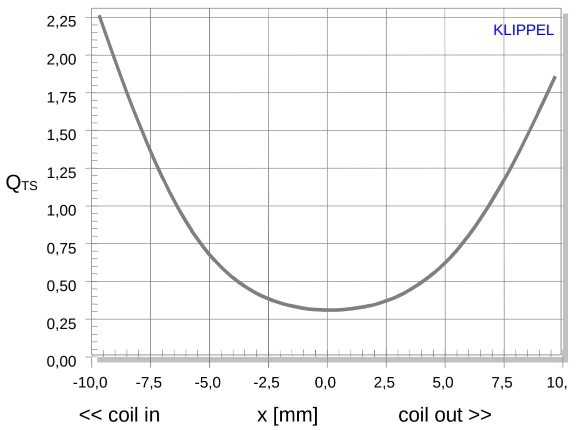

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Fig. 12: Total loss factor $Q_{TS}(x)$ versus displacement x

How the nonlinear *Bl*(x)-characteristic and the linear parameter estimates are related?

If the linear *Bl*-parameter is measured at sufficiently small amplitudes the estimate agrees very well with the value of the nonlinear characteristic *Bl(x=0)* at the rest position. However, if the linear parameter is estimated at medium or high amplitudes usually a smaller value is obtained. This is due to the fact that the linear parameter measurement calculates the mean value of the *Bl(x)*-variations weighted by the probability density function pdf*(x)* of the displacement for the particular excitation signal. For a noise-like excitation signal the voice coil is most of the time close to the rest position and the linear parameter *Bl* deviates only by 5-10 percent from the small signal value even the *Bl(x)* reduces down to 25 percent at peaks.

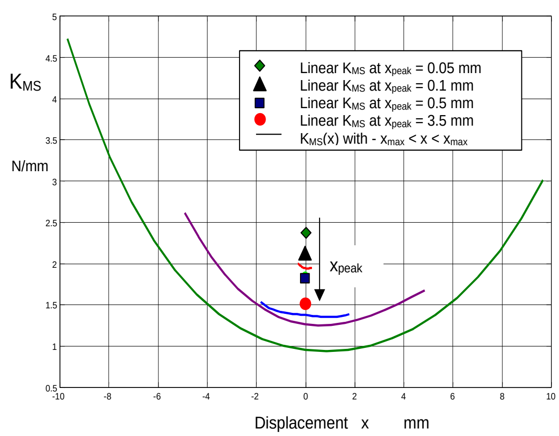

Fig. 13: Linear stiffness K_{MS} and nonlinear stiffness $K_{MS}(x)$ of mechanical suspension measured at different amplitude levels

Figure 13 shows the nonlinear stiffness curves $K_{MS}(x)$ measured at different amplitudes (x_{peak} = 0.3, 1.8, 5, 9 mm). As expected the curves show that the suspension becomes stiffer at high amplitudes. However, the curves measured at different amplitudes do not coincide as the *Bl(x)*-characteristic and other nonlinear parameters do. Instead the stiffness decreases with rising peak displacement x_{peak}. At maximal amplitudes x_{peak}= 9 mm the $K_{MS}(x=0)$ reduces to the half value of the stiffness measured at the smallest amplitude x_{peak}= 0.3 mm. This agrees with the results of the linear parameter measurement represented in figure 13 as symbols. Increasing the excitation signal by 20 dB the stiffness reduces from 2.3 N/mm at x_{peak} = 0.05 mm to 1.8 N/mm at x_{peak} = 0.5 mm.

Apparently, a high displacement changes the geometry of the fibres of the suspension and reduces the stiffness of the total arrangement. If the coil returns to the rest position the deformation will still persist for some time due to the viscous properties of the used materials. Since this phenomenon can be found on most drivers additional research is required to establish a more precise model of this complicated mechanism.

With the current knowledge we summarize that the dynamic behaviour suspension depends not only on the instantaneous displacement but also on the peak value x_{peak} occurred in the last period of time. The first dependency explains the increase of the stiffness at high amplitudes. The second explains the loss of stiffness near the rest position. Clearly both mechanisms are nonlinear but the second one starts already in the small signal domain.

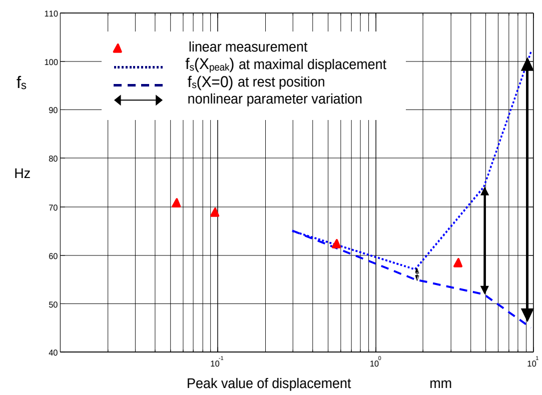

Fig. 14: Resonance frequency f_s of the driver versus peak value of displacement

Due to the variation of the suspension parameters also the resonance frequency depends on the displacement. Figure 12 shows the instantaneous resonance frequency f_s as a function of the peak displacement x_{peak}. Triangles represent the results of linear parameter measurements performed at four different amplitudes. The dashed and the dotted line show the range of variation of $f_s(x)$ versus instantaneous displacement $-x_{peak} < x < x_{peak}$ measured using the nonlinear identification technique. In the small signal domain both measurements coincide and confirm the decrease of the resonance frequency with rising peak displacement. It is interesting to see that even at very small amplitudes the resonance frequency is not a constant.

Whereas at low amplitudes the variations of $f_s(x)$ due to instantaneous displacement are small, above x_{peak}= 2mm the dotted line (resonance frequency f_s at maximal displacement) rises rapidly while the dashed line (resonance frequency f_s at rest position) follows the tendency to lower values. This may be considered as the begin of the large signal domain. At x_{peak}= 9 mm the instantaneous variations of $f_s(x)$ exceed more than one octave. The linear parameter measurement can reflect only a mean value of $f_s(x)$ that depends on the probability density function of the displacement.

Summary

A fast, one step algorithms was described that identifies the components of the linear loudspeaker model shown in figure 1. As an second (perturbed) measurement is avoided accurate results with higher reproducibility are obtained. The algorithm exploits the voltage to displacement transfer function that can be measured using a laser displacement sensor. The transfer function is affected by the suspension creep at low frequencies. A proper model can be obtained only if the stiffness is modeled as an frequency varying parameter. Note that the electrical impedance gives virtually no information about the properties of the suspension at low frequencies. As the identification results are worthless without a proper validity check a validation procedure was proposed. It uses some additional information contained in the measured signals.

It was pointed out that despite linear parameters are straight forward and appealing they are meaningless in the large signal domain. As some parameters (see figure 13) are not constant and vary even at small amplitudes the question arises: "What is the small signal domain ?". Linear modeling of the mechanical suspension seems to be an inadequate simplification of the reality. The dynamic behaviour of the suspension depends not only on the instantaneous displacement but also on the peak displacement occurred in the last period of time. Although it is convenient and common practice to express compliance C_{MS} and resonance frequency f_s by single numbers this information shows only a small part of the whole picture. The "single number" linear parameters can be interpreted as weighted mean values of the corresponding nonlinear parameter curves.

However there is no contradiction between linear and large signal parameters. There is in fact a smooth connection as large signal parameters preserve and generalize the linear parameters. The large signal parameters give additional information valuable for assessing the permissible working range (maximal displacement, maximal power) and the permissible mechanical and thermal load. Furthermore the large signal parameters reveal the dominant sources of distortion.

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