Auralization of Signal Distortion in Audio Systems
Part 2: Transducer Modeling

Wolfgang Klippel, Klippel GmbH

A new method is presented for the auralization of selected distortion components generated by regular nonlinearities inherent in loudspeaker systems. Contrary to the generic approach presented in the first part the alternative approach presented here exploits the results of lumped parameter modeling in the state space. A mixing device generates a virtual output signal comprising nonlinear distortion attenuated or enhanced by a user-defined scaling factor. The auralization output can be used for systematic listening tests or perceptive modeling to determine audibility thresholds and to assess the impact on sound quality of the dominant nonlinearities in loudspeakers.

1. INTRODUCTION

This paper continues the discussion on the auralization of signal distortion but sets the focus on techniques, which exploit available information on physical modeling of the regular nonlinearities. Transducers and loudspeaker systems operated in the small signal domain can be modelled at sufficient accuracy by linear models using lumped or distributed parameters. The relationship between electrical input and acoustical output or any other state variable such as displacement x can be described by a complex transfer function in the frequency domain or an impulse response in the time domain. At higher amplitudes the transducer behaves nonlinearly and generates additional distortion in the output signal. Dominant nonlinearities of the transducer which are related to motor, suspension and enclosure can be reliably modeled by a nonlinear network comprising lumped parameters.

The difference auralization technique [1] can use the linear model as the desired reference system and the nonlinear model for representing the device under test (DUT) as shown in Fig. 1.

A separator adjusts the gain and time delay of the linear reference signals x_R to the output x_T of the nonlinear model and generates a difference signal p_d which represents the nonlinear distortion of the transducer. A following mixer generates the auralization output p_A by adding the distortion p'_d scaled by the auralization gain S_{dis} to the linear reference signal. For S_{dis}=0 and S_{dis}=1 the auralization output p_A corresponds with the output signals of the linear and nonlinear model, respectively, while for S_{dis}>1 the distortion can be arbitrarily enhanced. This approach requires that the linear and nonlinear models are identical in the small signal domain. Any discrepancy will generate a difference signal which comprises a residual of the linear signal which will be interpreted as distortion.

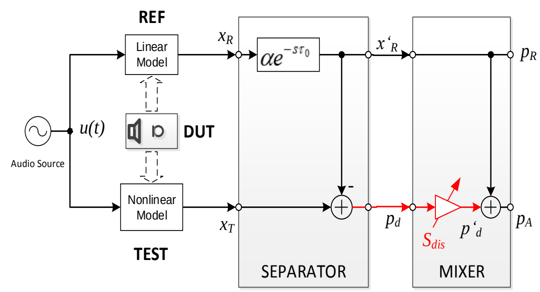

Fig. 1 : Auralization of regular nonlinear distortion based on the difference technique [1] using a linear and a nonlinear model of the device under test (DUT)

The difference technique can only be used for auralization of the total nonlinear distortion. However, the multitude of nonlinearities inherent in the transducer spark an interest in evaluating the corresponding distortion components separately [3].

Although this is the main objective here the paper starts with the reformulation of the difference technique based on a state space model and uses this framework later for the auralization of distortion components.

2. TOTAL DISTORTION

2.1. Difference Output Auralization

The electro-acoustical transducer under test is modeled as a nonlinear system (TEST) having a single input but multiple outputs (SIMO). The one-dimensional signal path from the electrical input signal $u(t)$ to the volume velocity $q(t)$ is represented by a network comprising lumped elements with linear and nonlinear parameters [2] as shown for a vented box system in Fig. 9 in the appendix. The prediction of the volume velocity q assumes an effective radiation area S_D which is frequency independent. A linear system with the transfer function $H_{rad}(\mathbf{r},s)$ represents mechanical vibration after cone break up, sound radiation and propagation of the wave to the particular observation point **r**. The voice coil displacement x, current i and other state variables of the lumped parameter model can be summarized into a state vector $\mathbf{x}= [x_1, ..., x_j , ..., x_J]^T$ fulfilling the differential equation in state space form

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(\mathbf{x})u \tag{1}$$

where the matrix $\mathbf{A}(\mathbf{x})$ and vector $\mathbf{B}(\mathbf{x})$ are nonlinear functions of the state vector **x**. The volume velocity

$$q(t) = \mathbf{C}(\mathbf{x})\mathbf{x} + D(\mathbf{x})u \tag{2}$$

is calculated from the state vector **x** and voltage u by using the scalar $D(\mathbf{x})$ and vector $\mathbf{C}(\mathbf{x})$ which are in general also nonlinear functions of state vector **x**.
Eq. (1) causes a feedback path in the nonlinear pole model (NPM) in Fig. 2 while Eq. (2) corresponds to the following feed-forward path.

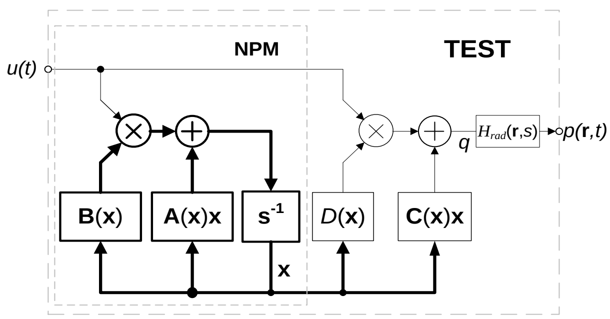

Fig. 2 : Nonlinear state space model (TEST) of the electro-acoustical transducer under test

The sound pressure output

$$p(\mathbf{r},t) = L^{-1}\{H_{rad}(\mathbf{r},s)\} * q(t) \tag{3}$$

is generated by the convolution of the volume velocity q with the impulse response derived from the transfer function $H_{rad}(\mathbf{r},s)$ by using the inverse Laplace transform. The coefficients $\mathbf{A}(\mathbf{x})$, $\mathbf{B}(\mathbf{x})$, $\mathbf{C}(\mathbf{x})$ and $D(\mathbf{x})$ and the transfer function $H_{rad}(\mathbf{r},s)$ are specified for a vented-box loudspeaker system in the appendix.
The reference system (REF) is modelled as a linear system as illustrated in Fig. 3 corresponding to

$$\dot{\mathbf{x}}_0 = \mathbf{A}(\mathbf{0})\mathbf{x}_0 + \mathbf{B}(\mathbf{0})u \tag{4}$$

$$q_0(t) = C(\mathbf{0})\mathbf{x}_0 + D(\mathbf{0})u \tag{5}$$

$$p_0(\mathbf{r},t) = L^{-1}\{H_{rad}(\mathbf{r},s)\} * q_0(t) \tag{6}$$

using coefficients which are constants describing the transducer in the small signal domain for $\mathbf{x}=\mathbf{0}$.

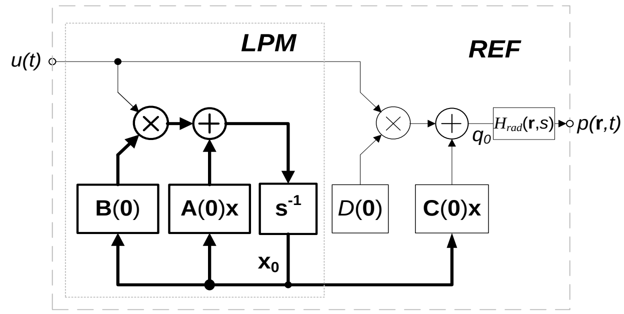

Fig. 3 : Linear state space model (REF) of the electro-acoustical transducer under test

The auralization output signal is the virtual sound pressure

$$\begin{aligned} p_A(t) &= p_0(t) + S_{dis}\, p_d(t) \\ &= p_0(t) + S_{dis}\left(p(t) - p_0(t)\right) \end{aligned} \tag{7}$$

in which the total distortion p_d can be arbitrary scaled by S_{dis}.

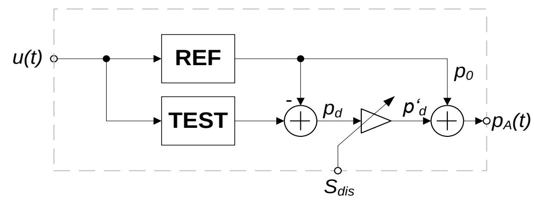

Fig. 4 : Difference auralization technique using a linear transducer model (REF) and a nonlinear transducer model (TEST).

Contrary to the generic approach in Fig. 1 auralization based on transducer modeling dispenses with any gain and time delay adjustment of the test signal $p(t)$ to the reference signal $p_0(t)$ in Fig. 4.

2.2. Difference State Auralization

Modeling of the test and reference system makes it possible to separate the distortion also in the state vector

$$\mathbf{x}_d = \mathbf{x} - \mathbf{x}_0 \tag{8}$$

by subtracting the state vectors $\mathbf{x}_0$ provided from the linear pole model (LPM) from the state vector **x** from the output of the nonlinear pole model (NPM) as shown in Fig. 5.

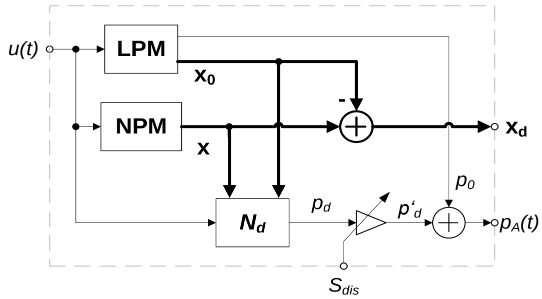

Fig. 5 : Difference state auralization of the total nonlinear distortion

The distortion vector $\mathbf{x}_d$ can be calculated by the differential equation

$$\begin{aligned}\dot{\mathbf{x}}_d &= \dot{\mathbf{x}} - \dot{\mathbf{x}}_0 \\ &= \mathbf{A}(\mathbf{x})\mathbf{x} - \mathbf{A}(\mathbf{0})\mathbf{x}_0 + \left(\mathbf{B}(\mathbf{x}) - \mathbf{B}(\mathbf{0})\right)u \\ &= \mathbf{A}_d(\mathbf{x})\mathbf{x} + \mathbf{A}(\mathbf{0})\mathbf{x}_d + \mathbf{B}_d(\mathbf{x})u\end{aligned} \tag{9}$$

with

$$\begin{aligned}\mathbf{A}_d(\mathbf{x}) &= \mathbf{A}(\mathbf{x}) - \mathbf{A}(\mathbf{0}) \\ \mathbf{B}_d(\mathbf{x}) &= \mathbf{B}(\mathbf{x}) - \mathbf{B}(\mathbf{0})\end{aligned}. \tag{10}$$

In the same way the distortion in the volume velocity

$$\begin{aligned}q_d(t) &= q(t) - q_0(t) \\ &= \mathbf{C}(\mathbf{x})\mathbf{x} - \mathbf{C}(\mathbf{0})\mathbf{x}_0 + \left(D(\mathbf{x}) - D(0)\right)u \\ &= \mathbf{C}_d(\mathbf{x})\mathbf{x} + \mathbf{C}(\mathbf{0})\mathbf{x}_d + D_d(\mathbf{x})u\end{aligned} \tag{11}$$

can be calculated by using the input voltage u, the state vectors $\mathbf{x}_d$, $\mathbf{x}$, $\mathbf{x}_0$ and the coefficients in

$$\begin{aligned}\mathbf{C}_d(\mathbf{x}) &= \mathbf{C}(\mathbf{x}) - \mathbf{C}(\mathbf{0}) \\ D_d(\mathbf{x}) &= D(\mathbf{x}) - D(\mathbf{0})\end{aligned}. \tag{12}$$

The linear filtering of the volume velocity distortion q_d gives the total distortion in the sound pressure signal

$$p_d(\mathbf{r},t) = L^{-1}\{H_{rad}(\mathbf{r},s)\} * q_d(t)\,. \tag{13}$$

Fig. 6 illustrates Eqs. (10), (11) and (13) as a signal flow chart, where a feedback path generates the same poles as the linear pole model (LPM).

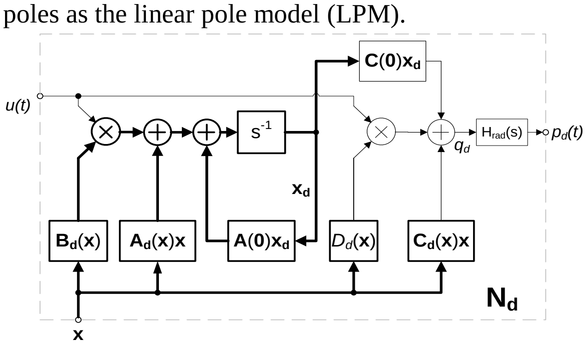

Fig. 6 : Generation of the total nonlinear distortion p_d based on the difference state vector $\mathbf{x}_d$.

3. DISTORTION COMPONENTS

The difference state auralization reveals how all nonlinearities inherent in the transducer affect the state variables. However, the auralization of the total output distortion can already be accomplished by using the simple approach in Fig. 4. The mathematical effort and the increased complexity of the model pays back when this approach is used for separating distortion components in the acoustical output p and using them for auralization.

There are many ways to decompose the total distortion p_d into meaningful components. The following decompositions are of particular interest:

- Separating the distortion generated by each nonlinear parameter such as force factor $Bl(x)$, stiffness $K_{ms}(x)$, inductance $L(x)$ ...
- Separating multiple effects generated by a single nonlinearity such as the nonlinear electrical damping effect corresponding to $Bl(x)^2v/R_e$ from parametric excitation effect in driving force $Bl(x)i$ wherein both effects are caused by nonlinear force factor $Bl(x)$.
- Separating distortion generated by odd-order terms in the power series expansion of the nonlinear parameter from distortion generated by even-order terms.
- Separating distortion generated by low-order terms from components caused by high-order terms in the power series expansion of the nonlinear parameter.

The common point of all decomposition schemes is the separation of the coefficients $\mathbf{A}_d(\mathbf{x})$, $\mathbf{B}_d(\mathbf{x})$, $\mathbf{C}_d(\mathbf{x})$, $\boldsymbol{D}_d(\mathbf{x})$ according to

$$\begin{aligned}\mathbf{A}_d(\mathbf{x}) &= \sum_{n=1}^{N}\mathbf{A}_n(\mathbf{x}) \\ \mathbf{B}_d(\mathbf{x}) &= \sum_{n=1}^{N}\mathbf{B}_n(\mathbf{x}) \\ \mathbf{C}_d(\mathbf{x}) &= \sum_{n=1}^{N}\mathbf{C}_n(\mathbf{x}) \\ D_d(\mathbf{0}) &= \sum_{n=1}^{N}D_n(\mathbf{x})\end{aligned} \tag{14}$$

in a sum of N auralization coefficients wherein nth coefficients $\mathbf{A}_n(\mathbf{x})$, $\mathbf{B}_n(\mathbf{x})$, $\mathbf{C}_n(\mathbf{x})$, $D_n(\mathbf{x})$ generate a contribution $\mathbf{x}_n$ to the total distortion of the state vector

$$\mathbf{x}_d = \sum_{n=1}^{N}\mathbf{x}_n\,, \tag{15}$$

a contribution q_n to the total distortion in the volume velocity

$$q_d = \sum_{n=1}^{N}q_n \tag{16}$$

and a contribution p_n to the total distortion p_d sound pressure

$$p_d = \sum_{n=1}^{N} p_n \,. \tag{17}$$

Inserting Eqs. (14) and (15) in Eq. (9) gives

$$\sum_{n=1}^{N} \dot{\mathbf{x}}_n = \sum_{n=1}^{N} \mathbf{A}_n(\mathbf{x})\mathbf{x} + \mathbf{A}(\mathbf{0})\left(\sum_{n=1}^{N} \mathbf{x}_n\right) + \sum_{n=1}^{N} \mathbf{B}_n(\mathbf{x})u \tag{18}$$

which can be separated in N systems wherein each system generates distortion component $\mathbf{x}_n$ of the state vector by the differential equation:

$$\dot{\mathbf{x}}_n = \mathbf{A}_n(\mathbf{x})\mathbf{x} + \mathbf{A}(\mathbf{0})\mathbf{x}_n + \mathbf{B}_n(\mathbf{x})u \qquad n = 1,...,N \tag{19}$$

Inserting Eqs. (14) and (16) in Eq. (11) gives

$$\sum_{n=1}^{N} q_n(t) = \sum_{n=1}^{N} \mathbf{C}_n(\mathbf{x})\mathbf{x} + \mathbf{C}(\mathbf{0})\left(\sum_{n=1}^{N} \mathbf{x}_n\right) + \sum_{n=1}^{N} D_n(\mathbf{x})u \tag{20}$$

which can be also separated N systems with

$$q_n = \mathbf{C}_n(\mathbf{x})\mathbf{x} + \mathbf{C}(\mathbf{0})x_n + D_n(\mathbf{x})u \qquad n = 1,...,N \tag{21}$$

generating the distortion component of the volume velocity q_n. Post filtering of distortion component q_n gives the distortion component in the sound pressure output

$$p_n(\mathbf{r},t) = L^{-1}\{H_{rad}(\mathbf{r},s)\} * q_n(t) \,. \tag{22}$$

The synthesis of the distortion component based on the state vectors $\mathbf{x}$ and $\mathbf{x}_0$ is illustrated as a signal flow chart in Fig. 7**.**

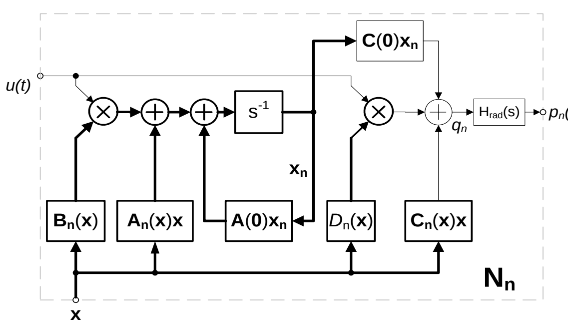

Fig. 7 : Partial distortion synthesis system generating the output distortion component p_n

For a vented-box loudspeaker system the decomposition of the total distortion into seven components (N=6) corresponding to the dominant nonlinearities found in vented-box systems is presented in the appendix. The matrix $\mathbf{A}_n(\mathbf{x})$ and vectors $\mathbf{B}_n(\mathbf{x})$ and $\mathbf{C}_n(\mathbf{x})$ are sparsely occupied and comprise only one dominant nonlinear parameter variation.

The distortion components p_n at the output of the partial distortion synthesis system N_n can be used for generating a virtual auralization output

$$p_A(t) = p_0(t) + \sum_{n=1}^{N} S_n p_n(t) \tag{23}$$

using an arbitrary scaling factor S_n for the nth component.

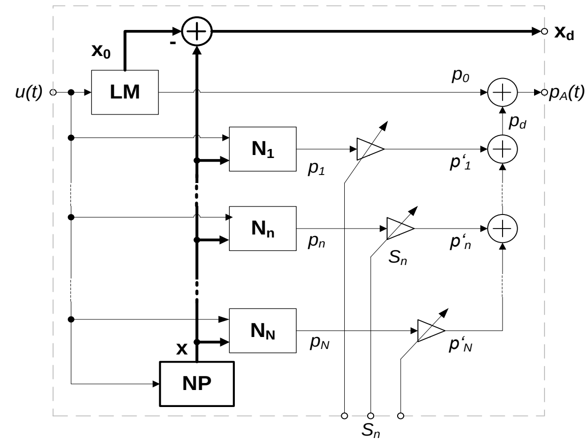

Fig. 8 : Auralization of the distortion components using partial distortion synthesis systems $\mathbf{N}_n$

The signal flow chart in Fig. 8 illustrates the resulting auralization scheme. The nonlinear state vector $\mathbf{x}$ generated by the nonlinear pole model (NP) according to Eq. (1) and the linear state vector $\mathbf{x}_0$ generated by the linear model (LM) according to Eq. (4) are supplied to all synthesis systems N_n. All nonlinearities inherent in the transducer affect the nonlinear state vector $\mathbf{x}$ and indirectly the distortion generation in all synthesis systems N_n with n=1,...,N. However, each synthesis system N_n describes the primary contribution of each nonlinearity to the total distortion while considering the influence of the other nonlinearities via the state vector $\mathbf{x}$ in the feed-back loop. Thus, it is useful to distinguish between a primary and a secondary effect of each nonlinearity. For example an asymmetric stiffness characteristic in a woofer may generate a DC-component in displacement and other low frequency distortion. The auralization may reveal low audibility and no annoyance of primary distortion. However, the dynamically generated dc-displacement moves the coil from the optimal rest position generating significant intermodulation distortion in the audio band. The nonlinear force factor Bl(x) in this case generates a secondary effect while the asymmetrical stiffness of the mechanical suspension is the root cause of the problem. The synthesis system N_n and the following scaling by S_n can be used to auralize the second effect. Using identical values S_n=S_{dis} in all scaling devices with n=1,...,N an auralization of the total distortion p_d can be accomplished which is identical with the difference technique in chapter 2. The scaling factors S_n have no influence on the internal state vector $\mathbf{x}$.

4. OBJECTIVE ASSESSMENT

The combination of transducer modeling and the new decomposition technique open new ways for assessing the distortion from a physical perspective. Short-term and long-term spectral analysis may be applied to the distortion components in the state vector **x** and in the sound pressure output *p*. Single-valued characteristics evaluating the rms-value or peak value of the distortion can be displayed versus time to find critical stimuli exciting the nonlinearities inherent in the transducer. The following ratios have been proven useful in practical applications:

4.1. Total Distortion Ratio

The Total Distortion Ratio defined by

$$TDR(t) = \frac{\underset{T}{Max}\left|S_{dis} p_d(t)\right|}{\underset{T}{Max}\left|p_A(t))\right|} 100\% \qquad (24)$$

and

$$TDR(t) = \frac{\underset{T}{Max}\left|\sum_{n=1}^{N} S_n p_n(t)\right|}{\underset{T}{Max}\left|p_A(t))\right|} 100\% \qquad (26)$$

for the auralization of the total distortion and components, respectively, describes the ratio between the peak values of the distortion and the auralization output $p_A(t)$ within the time frame t and $t+T$. The total distortion ratio TDR(t) considers in contrast to total harmonic distortion (THD) all distortion components including harmonics and intermodulation components.

4.2. Partial Distortion Ratio

The contribution of each nonlinearity to the total auralization output $p_A(t)$ can be described by the objective metric

$$DR_n(t) = \frac{\underset{T}{Max}\left|S_n p_n(t))\right|}{\underset{T}{Max}\left|p_a(t)\right|} 100\% \qquad n = 1,\ldots,N \qquad (28)$$

considering the peak values of the nth distortion component and total signal. This ratio is useful for detecting significant nonlinearities generating dominant distortion in the acoustical output.

4.3. State Distortion Ratio

In order to understand the interaction between multiple nonlinearities and the root cause of distortion a state distortion ratio was defined as

$$SDR(t,n,j) = \frac{\underset{T}{Max}\left|x_n[j]\right|}{\underset{T}{Max}\left|x[j]\right|} 100\% \qquad \begin{matrix} 1 \le n \le N \\ 1 \le j \le J \end{matrix} \qquad (30)$$

which is the peak ratio of the nth distortion component and the j^{th} state variable. This ratio can be displaced as a matrix with the dimensions N x J.

A high value of $SDR(t,n,j)$ indicates that the nth distortion component may be a root cause of a secondary distortion generation process caused by other nonlinearities which depend on the jth state variable.

5. DISCUSSION

The model-based auralization technique dispenses with measurements of state variables in a device under test and a reference unit but requires linear and nonlinear parameters used in the modeling. This input information can be provided by system identification techniques applied to existing transducers or systems. In an early design stage the linear and nonlinear parameters of a virtual prototype can be derived from other kinds of numerical simulations such as FEA and BEA.

This method is very powerful for nonlinearities in the motor and suspension system where a reliable nonlinear lumped parameter model is available. At the moment this auralization technique is not applicable to nonlinear distortion related to cone break-up due to lack of an accurate model describing the nonlinear vibration at higher frequencies where distributed parameters and a multitude of state variables are required [4]. It is very unlikely that the model-based technique gains significant importance for assessing rub & buzz and other irregular distortion caused by loudspeaker defects [5]. Distortions which have a random nature such as loose particles are difficult to model and can be auralized by the difference technique based on the measurement of a test and a reference system.

6. CONCLUSION

Auralization of signal distortion combines perceptive and physical assessment of the audio system. The new decomposition technique allows to evaluate any transducer nonlinearity separately. Distortion components which are most likely masked by other signal components can be virtually enhanced to determine their audibility threshold and their impact on perceived sound quality [6]. This is an ideal basis for setting up systematic listening tests to find the dominant source of distortion and to optimize the design for highest performance-cost ratio. For example, the distortion generated by nonlinear force factor $Bl(x)$ and inductance $L(x)$ can be compared to decide whether the coil-gap configuration needs an additional shorting ring in the particular application.

The model-based technique presented in this paper gives full transparency of the distortion generation process including the role of the stimulus, the contribution of each nonlinearity and the transfer of the distortion to the listening point. This simplifies the loudspeaker diagnostics to find the root cause of problem.

7. REFERENCES

[1] W. Klippel, "Auralization of Signal Distortion in Audio Systems, Part 1: Generic Modeling" 51st International Conference: Loudspeakers and Headphones, Helsinki, Finland (August 2013).

[2] W. Klippel, "Loudspeaker Nonlinearities – Causes, Parameters, Symptoms," J. Audio Eng. Society, Vol. 54, No. 10, pp. 907-939, October 2006

[3] W. Klippel, "Speaker Auralization –Subjective Evaluation of Nonlinear Distortion," presented at the 110th Convention of the Audio Eng. Society, Amsterdam, May 12-15, 2001, preprint 5310, J.Audio Eng. Society, Vol. 49, No. 6, 2001 June, p. 526.

[4] N. Quaegebeur, "Mechanical Resonance and Geometrical Nonlinearities in Electrodynamic Loudspeakers," J. Audio Eng. Society, Vol. 56, No. 6, pp. 462-472, June 2008.

[5] W. Klippel, "Measurement of Impulsive Distortion, Rub and Buzz and other Disturbances," Presented at the 114th convention of the Audio Eng. Soc. March 1st, 2003, preprint 5734.

[6] C. Kaernbach,"Simple adaptive testing with the weighted up-down method," Perception & Psyhcoacoustics, 1991, 49 (3), pp. 227-229.

8. APPENDIX

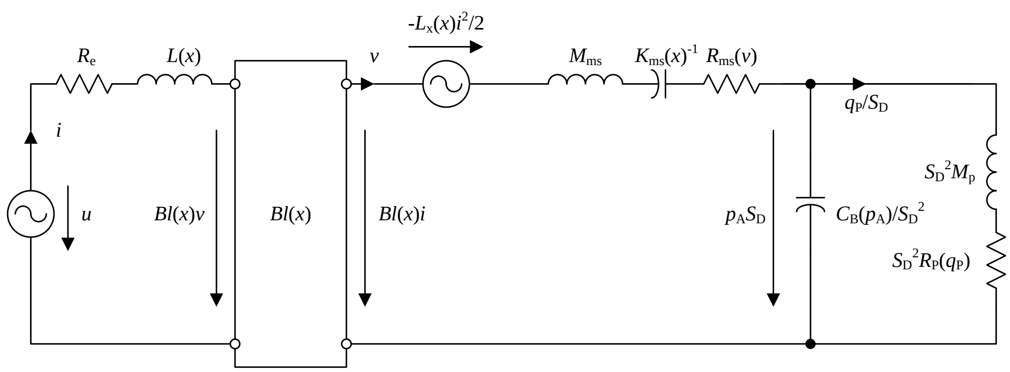

Fig. 9 : Electrical network model representing a vented box loudspeaker system

8.1. Lumped Parameter Modeling

A vented-box loudspeaker system is modeled by the equivalent circuit in Fig. 9 using the following state variables:

u	electrical terminal voltage,
i	electrical input current,
x	voice coil displacement,
v	voice coil velocity,
p_A	sound pressure in the vented enclosure,
q_p	volume velocity in the port,

and lumped parameters:

R_e	electrical dc-resistance of the voice coil,
$L(x)$	voice coil inductance as a function of voice coil displacement x,
$L_x(x)$	local derivative of the voice coil inductance with respect to displacement x,
$Bl(x)$	force factor as a function of voice coil displacement x,
M_{ms}	total moving mass including air load,
$K_{ms}(x)$	stiffness of mechanical suspension as a function of voice coil displacement x,
$R_{ms}(v)$	mechanical resistance as a function of voice coil velocity v,
$C_B(p_A)$	acoustical compliance of the air in the vented enclosure as a function of the sound pressure p_A,

S_D effective radiation area of the radiator
M_P acoustical mass of the moving air in the port
$R_P(q_P)$ acoustical resistance representing the losses of the air in the port as a function of the volume velocity q_P.

8.1. Nonlinear State Space Model

Summarizing the state variables in the state vector $\mathbf{x}=[x_1, x_2, x_3, x_4, x_5]^T = [x, v, i, q_P, p_A]^T$ the coefficients of the state space model in Eqs. (1) and (2) are

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{K_{ms}(x_1)}{M_{ms}} & -\frac{R_{ms}(x_2)}{M_{ms}} & \frac{Bl(x_1)}{M_{ms}} + \frac{L_x(x_1)x_3}{2M_{ms}} & 0 & -\frac{S_D}{M_{ms}} \\ 0 & -\frac{Bl(x_1)+L_x(x_1)x_3}{L(x_1)} & -\frac{R_e}{L(x_1)} & 0 & 0 \\ 0 & 0 & 0 & -\frac{R_P(x_4)}{M_P} & \frac{1}{M_P} \\ 0 & \frac{S_D}{C_B(x_5)} & 0 & -\frac{1}{C_B(x_5)} & 0 \end{bmatrix}$$

$$\mathbf{B}(\mathbf{x}) = \begin{bmatrix} 0 & 0 & \frac{1}{L(x_1)} & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{C}(\mathbf{x}) = \begin{bmatrix} 0 & S_D & 0 & -1 & 0 \end{bmatrix}$$

$$D(\mathbf{x}) = [0]$$

and the transfer function in Eq. (3)

$$H_{rad}(\mathbf{r}, s) = -\frac{\rho}{2\pi} s \frac{e^{-sr/c}}{r}$$

is modelled by a point source radiating the sound into the half space with the density of air ρ, distance r and speed of sound c.

8.2. Auralization Model

Auralization matrix of the mechanical stiffness $K_{ms}(x)$:

$$\mathbf{A}_1(\mathbf{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{K_{ms}() - K_{ms}(x_1)}{M_{ms}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Auralization matrix of the force factor $Bl(x)$:

$$\mathbf{A}_2(\mathbf{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{Bl(x_1) - Bl(0)}{M_{ms}} & 0 & 0 \\ 0 & \frac{Bl(0)}{L(0)} - \frac{Bl(x_1)}{L(0)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Auralization matrix of the mechanical resistance $R_{ms}(v)$:

$$\mathbf{A}_3(\mathbf{x}) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{R_{ms}(0) - R_{ms}(x_2)}{M_{ms}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Auralization matrix of the acoustic resistance $R_P(q_P)$:

$$\mathbf{A}_4(\mathbf{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{R_P(0)}{M_P} - \frac{R_P(x_4)}{M_P} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Auralization matrix of the acoustic compliance $C_B(p_A)$:

$$\mathbf{A}_5(\mathbf{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{S_D}{C_B(x_5)} - \frac{S_D}{C_B(0)} & 0 & \frac{1}{C_B(0)} - \frac{1}{C_B(x_5)} & 0 \end{bmatrix}$$

Auralization matrices of the electrical inductance $L(x)$:

$$\mathbf{A}_6(\mathbf{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{L_x(x_1)x_3}{2M_{ms}} & 0 & 0 \\ 0 & \frac{Bl(x_1)}{L(0)} - \frac{Bl(x_1)}{L(x_1)} + \frac{L_x(x_1)x_3}{L(x_1)} & \frac{R_e}{L(0)} - \frac{R_e}{L(x_1)} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_6(\mathbf{x}) = \begin{bmatrix} 0 & 0 & \frac{1}{L(x_1)} - \frac{1}{L(0)} & 0 & 0 \end{bmatrix}^T$$

else

$$\mathbf{B}_n(\mathbf{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \qquad n = 1,\ldots,5$$

$$\mathbf{C}_n(\mathbf{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad n = 1,\ldots,6$$

$$D_n(\mathbf{x}) = [0] \qquad n = 1,\ldots,6$$